Special Relativity First Year Notes (MT)

Introduction

Frame of reference: The coordinate system in which we have chosen to make our measurements

Inertial frames of reference: - Newton's Laws are obeyed (particularly 1st Law → frame not accelerating)

- all IFRs are in uniform motion wrt one another

Einstein's Postulates → 1. = The Laws of Physics are the same in all inertial frames

2. = Light travels in a vacuum with the same speed C, at all times, in all directions in all IFRs independently of the motion of the emitter

Observer = (in an IFR) the whole collection of synchronised clocks deemed to be measuring time at their locations throughout all IFR

Frame coordinates = conceptually we imagine that an IFR is 'pre-coordinated' → recorded wrt an origin

Events = an instantaneous point-like occurance (eg. particle collision, flash of light, tick of clock)

Proper time = The 'shortest' interval between two events measured in the frame in which they are at rest (known as the "rest frame")

Proper length = The length of a body measured by an observer in an IFR in which the body is at rest (aka "rest length")

The space-time diagram

→ stationary objects map to vertical lines

→ Convention is time (ct) is vertical axis and x is horizontal axis

→ Ray of light shown as a diagonal line at $45°$

Lorentz transformations

$$t' = \gamma\left(t - \frac{\beta x}{c}\right)$$
$$x' = \gamma(x - c\beta t)$$
$$y' = y$$
$$z' = z$$

$$\Rightarrow \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

[there should/can be Δ infront of all relevant things]

if you're dealing with a simple length contraction/time dilation from/to a rest frame then just figure it out and $\times$ or $\div$ by γ $(\gamma > 1)$

inverse Lorentz transformations

$$t = \gamma\left(t' + \frac{\beta x'}{c}\right)$$
$$x = \gamma(x' + c\beta t')$$
$$y = y'$$
$$z = z'$$

$$\Rightarrow \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Minkowski Space-time

- In 1907, Minkowski took the 3D picture of an inertial frame described by xyz triads (3-vectors) and considered instead 4D space-time whose points $[ct, x, y, z]$ correspond to events
- (ct, x, y, z) have the dimensions of length (sometimes written as $(ct, \underline{r})$) → written as $x^{\mu} = (ct, \underline{r})$
  - ct: timelike component; $\underline{r}$: spacelike component
- Known as a "4-vector" and we can use it
- Define in Minkowski space-time the dot product of 4-vectors

↳ $\boxed{(ct, \underline{r}) \cdot (ct, \underline{r}) = c^2t^2 - x^2 - y^2 - z^2}$ (VERY IMPORTANT → NOTE ± SIGNS)

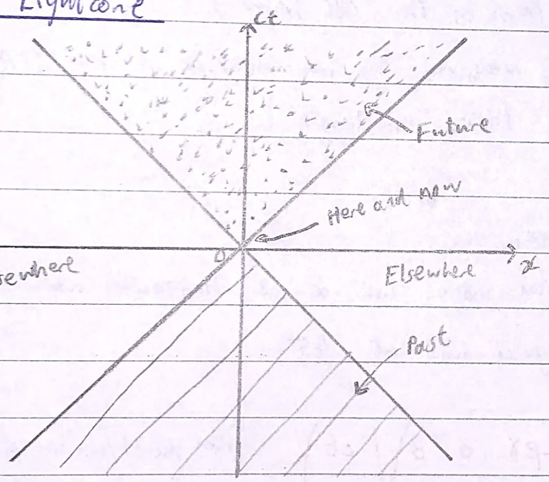
the interval → $s^2 \equiv c^2t^2 - x^2 - y^2 - z^2$ The interval is preserved over Lorentz trans. (the fact that this is conserved means that space and time cannot be separated)

Relativistic velocity addition: basically Lorentz transform for velocity

Consider an object moving with velocity V_x in frame S, what is its velocity V_x' in some different frame S'? [u = relative velocity between frames]

$$V_x' = \left(\frac{V_x - u}{1 - uV_x/c^2}\right), \quad V_y' = \frac{V_y}{\gamma(1 - uV_x/c^2)}, \quad V_z' = \frac{V_z}{\gamma(1 - uV_x/c^2)}$$

The Lightcone



Intervals inside the light cone

↳ $(\Delta s)^2 = c^2\Delta t^2 - \Delta x^2 > 0$: TIMELIKE

Intervals on the lightcone

↳ $(\Delta s)^2 = 0$: LIGHTLIKE

Intervals Elsewhere

↳ $(\Delta s)^2 < 0$: space-like

4-momentum (momenergy ":)")

$p^{\mu} = \left(\frac{E}{c}, \underline{p}\right)$, Relativistic momentum $= \underline{p} = \gamma m_0 \underline{v}$, Relativistic energy $= E = \gamma m_0 c^2$

Relativistic KE $= (\gamma - 1) m_0 c^2$ $(= E - m_0 c^2)$

For i^{th} particle: $p_i^{\mu} \cdot p_i^{\mu} = $ invariant

↱ ∴ conservation of total relativistic energy and momentum applies

For system: $\underbrace{\sum_i p_i^{\mu}}_{\text{before}} = \underbrace{\sum_j p_j^{\mu}}_{\text{after}}$ → $\underbrace{\sum_i E_i}_{\text{before}} = \underbrace{\sum_j E_j}_{\text{after}}$, $\underbrace{\sum_i p_{i,x}}_{\text{before}} = \underbrace{\sum_j p_{j,x}}_{\text{after}}$, $\underbrace{\sum_i p_{i,y}}_{\text{before}} = \underbrace{\sum_j p_{j,y}}_{\text{after}}$, same for z

(before and after: in the same frame)

across frames ↓ [$p_i^{\mu} \cdot p_i^{\mu}$ conserved; $\sum_i p_i^{\mu}$ conserved] ↑ in same frame

You can also use $p^{\mu}_{Tot} \cdot p^{\mu}_{Tot} = p^{\mu'}_{tot} \cdot p^{\mu'}_{tot}$ $[p^{\mu}_{tot} = \sum_i p_i^{\mu}]$

Collisions

Elastic collisions: KE is conserved → (rest energy and mass also conserved)

Explosive collisions: KE increases → (rest mass and energy decrease)

Sticky collisions: KE decreases → (rest mass and energy increase)

Momenergy cont.

We know $p^\mu \cdot p^\mu$ is invariant → $p^\mu \cdot p^\mu = \left(\frac{E}{c}, \underline{p}\right)\cdot\left(\frac{E}{c}, \underline{p}\right) = \frac{E^2}{c^2} - p^2 = m_0^2 c^2$ $\left[\begin{matrix} E = \gamma m_0 c^2 \\ p = \gamma m_0 v \end{matrix}\right]$

→ $E^2 = p^2c^2 + m_0^2c^4$ → a bit weird but useful in algebra to cancel

For photon: $E = pc$ $\left[\text{also } p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}\right]$ → $\boxed{\text{for photon } p^\mu \cdot p^\mu = 0}$

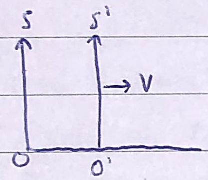
Compton effect: proved that photons are massless particles with momentum

Relativistic Doppler Effect

Source at origin O' in frame S' that flashes every τ' seconds (as measured in S').
What is the period of the flashing τ measured at O in frame S.

2 factors: ① Time dilation: τ' in $S' = \gamma\tau'$ in S

② Source moves to the right so light has to travel an extra distance $v\gamma\tau'$ to get to O, giving an extra time $\frac{v\gamma\tau'}{c}$ (② same in classical D.E)



Total time in S: $\tau = \gamma\tau' + \frac{v\gamma\tau'}{c}$

$= \gamma\tau'\left(1 + \frac{v}{c}\right)$

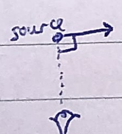
relativistic Doppler effect	$\tau = \tau'\sqrt{\frac{1+\beta}{1-\beta}}$
for frequency ν: $\left[\nu = \frac{1}{\tau}\right]$	$\nu = \nu'\sqrt{\frac{1-\beta}{1+\beta}}$
for wavelength λ: $\left[\lambda = \frac{c}{\nu}\right]$	$\lambda = \lambda'\sqrt{\frac{1+\beta}{1-\beta}}$

Positive velocities ($\beta > 0$) = O' receding from us
↳ For a receding source $\tau > \tau'$

receding: $\nu < \nu'$ (redshift)
approaching: $\nu > \nu'$ (blueshift)

Non-examinable: Transverse Doppler Effect

If source only moves perpendicular to our line of sight, then we only have the time dilation factor present.



$\tau = \gamma\tau'$

$\nu = \frac{1}{\gamma}\nu'$

$\lambda = \gamma\lambda'$

Einsteins Postulates

1st: The Laws [illegible]hysics are the same in all frames

2nd: The speed of light is the same in all inertial frames

Special Relativity

Speed of light same in all inertial frames

Space-time 4-vector

$$(ct, \underline{r})$$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$

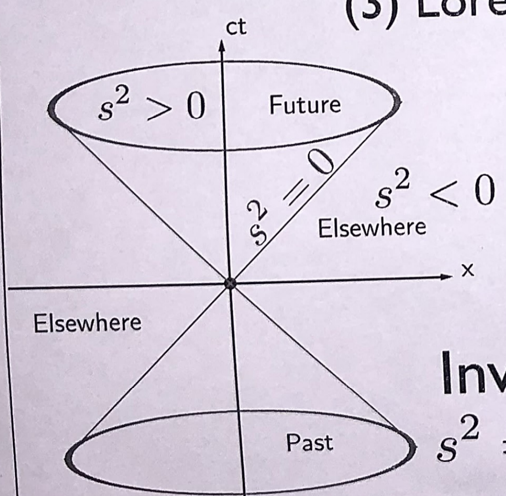
Lorentz transformation matrix

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

(1) ~~simultaneity~~

(2) Time dilation ← multiply or divide by γ

(3) Lorentz contraction



Energy 4-vector

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right)$$

Invariant:

$$E^2 = p^2c^2 + m_0^2c^4$$

$E = pc$ for a photon

KE $= (\gamma - 1) m_0 c^2$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 v$$

velocity addition

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2}$$

Invariant:

$$s^2 = c^2t^2 - r^2 = (ct, \underline{r}).(ct, \underline{r})$$

remember, dot product - signs

Doppler effect (longitudinal)

$$\tau = \tau' \sqrt{\frac{1+\beta}{1-\beta}}$$

period in rest frame of receding source